Towards Optimal Placement of Phasor Measurement Units for Smart Distribution Systems

Emad Jamil, Mohd. Rihan, *Senior Member, IEEE*, and M. A. Anees
Department of Electrical Engineering
Aligarh Muslim University
Aligarh-202002

***Abstract*—Phasor Measurement Units (PMUs) have potential role in power system monitoring, protection and control. PMUs directly measure and provide synchronized measurements of voltage and current phasors at various system locations. This offer numerous possibilities for ascertaining information related to the state and health of a power system.**

The aim of an optimal PMU placement is to provide minimal PMU installations to ensure complete observability of a power system. Several methods have been proposed in the past and significant work has been carried out till date to ensure observability of transmission networks. Penetration of new technologies at distribution level raises concern about system operation. These concerns can be addressed by synchrophasor applications. Hence, PMUs play an equally important role at distribution level also. Since, distribution systems are large in size as compared to transmission systems, the algorithms reported may take longer computational time for optimization. Hence, an approach towards optimal PMU placement for distribution system is required. This paper presents a simple and effective algorithm of PMU placement for distribution networks that saves considerable amount of computational time. The proposed algorithm is tested on IEEE-13, 34 and 123 Node Test Feeders and results show that computational time is considerably reduced without significantly affecting the number of PMUs.

Keywords—Phasor Measurement Units (PMUs);Optimal Placement of PMUs;Distribution Management System;Smart Grid

I. Introduction

Phasor Measurement Units (PMUs) become more and more important and attractive to power engineers because they can provide synchronized measurements of real-time phasors of voltage and currents [1]. Until recently, it was not possible to measure phase angle of the bus voltage in real time due to the technical difficulties in synchronizing measurements from distant locations. But introduction of the PMUs in power system has made it possible to measure the real-time phasors of voltages and currents at widely dispersed locations with respect to a global positioning system (GPS) clock [2].

For many years distribution networks have been the "poor cousin" of transmission networks in terms of computer-based monitoring equipment. Typically SCADA systems were considered essential for operation of transmission networks. Meanwhile frequently distribution networks were operated essentially manually by dispatching field crews to investigate customer report outages or to perform planned switching operations. Distribution systems are often designed in networked fashion (with isolation switches) but operated as radial system with main feeders extending from single distribution substation with limited sensors and communication.

However, emerging smart grid technologies are accelerating the transformation of the distribution system into a Smart Distribution System. Distribution systems are poised for significant changes as different types of loads, and small distributed generation sources are increasingly added to the system. Future distribution systems are likely to have higher penetrations of new and emerging technologies that are consistent with the objectives of the smart grid vision.

All the requirement of the objectives under smart grid vision and issues with traditional system rely on proper monitoring of the system. With the introduction of DER and active loads, things are likely to change. New dynamics are introduced which didn't existed before. This also requires improved monitoring and fast action control. Success is only possible if we can get the dynamic snapshot of the system at any instant. This situation would therefore justify the need for synchronized voltage and current phasor measurement in distribution grid.

PMU deployment may benefit some of the aspects of distribution systems mentioned below [3]:

- Stability analysis and monitoring
- Protection
- State estimation
- Voltage/ VAr / Watt control
- Fault location
- Loads with fast dynamics

Accurate phasor measurement can be achieved by deploying dedicated PMUs across the distribution grid. The new dynamics snapshot of the system can now be incorporated into Distribution Management System (DMS) in order to provide monitoring, control, energy management and protection schemes more suitable for this new situation.

For complete observability, each bus of the network has at least one phasor voltage measurement and one current phasor measurement. Placing a PMU on each bus of a system would not be economical. The cost and in turn the number of PMU to be installed must be minimized. Therefore, a methodology is needed to determine the optimal location of PMUs in a power system. The objective of the PMU placement problem is to find the minimum number of PMUs to make the system topologically observable, as well as the optimal locations of these PMUs. In recent year, there has been a significant research activity on the problem of finding the minimum number of PMUs and their optimal locations. In [4], a bisecting search method is implemented to find the minimum number of PMUs to make the system observable. The simulated annealing method is used to randomly choose the placement sets to the test for observability at each step of the bisecting search. In [1], the authors use a simulated annealing technique in their graph-theoretic procedure to find the optimal PMU locations. In [5] the authors use integer programming to determine the minimum number of PMUs. The method, however may suffer from the problem of being trapped in local minima. Multiple objectives, such as minimizing the measurement redundancy, cannot be handled by integer programming. Exhaustive search method for solving optimal PMU placement problem is discussed in [6]. In case of multiple solutions, a most preferable set based on measurement redundancy is selected. However, it is not suitable for large scale systems with huge search space. A novel topological method based on the augment incidence matrix and Tabu Search (TS) algorithm, is proposed in [7]. The solution of the combinatorial OPP problem requires less computation and is highly robust. The method is faster and more convenient than conventional observability analysis methods using complicated matrix analysis, because it manipulates integer numbers. A TS method on meter placement to maximize topological observability is presented in [7]. The GA method suggested in [8] solves the OPP problem using different PMU placement criteria, such as the absence of critical measurements and critical sets from the system, maximum quantity of measurements received as compared to the initial one, maximum accuracy of estimates, minimum cost of PMU placement, and transformation of the network graph into tree. The immune algorithm (IA) is a search strategy based on genetic algorithm principles and inspired by protection mechanisms of living organisms against bacteria and viruses.

From the previously reported algorithms in literature, Integer Programming method of PMU placement is found to be the most frequently used method for obtaining the optimal solution [6].

The size of distribution systems is generally large. The solution to placement problem of a large system takes a long time due to computational burden. Therefore, an optimal solution becomes very difficult to find & is difficult verify. Hence, an approach that provides optimal solution by reducing the computational time is required.

In this paper, an algorithm is proposed which provides solution in significantly short time compared to Integer Programming method. The paper is structured as follows: Section I presents a brief introduction. In Section II, the basic principle of PMU is described. In Section III, the problem of PMU placement is explained and new algorithm is proposed. Finally, Section IV & V conclude the findings.

II. PHASOR MEASUREMENT UNIT (PMU) TECHNOLOGY

Phasor Measurement Units (PMUs) are digital metering devices that use a Discrete Fourier Transform (DFT) algorithm in conjunction with a precisely timed GPS signal to provide synchronized phasor measurements at different locations in a power system. With GPS synchronization, this technology has the ability to synchronize measurements despite the large distances which may separate metering points. Fig. 1 shows the common elements of a PMU. Three phase voltages and currents coming from the power system are used as the input signals. After conversion to digital signals the PMU can calculate the phasors taking into account the time information. The device uses the pulse per second (PPS) as a synchronization impulse to start the measurements and an internal phase-locked-loop to provide impulses during one second before the next PPS. Each measurement coming out of PMU contains the UTC-time and date. This makes it possible to compare the measurements taken at exactly the same time at different locations in a network, which was not possible using non-synchronized measurements. Additionally, new parameters, like synchronized measured phase angle can be implemented and used for the state estimation issue [3].

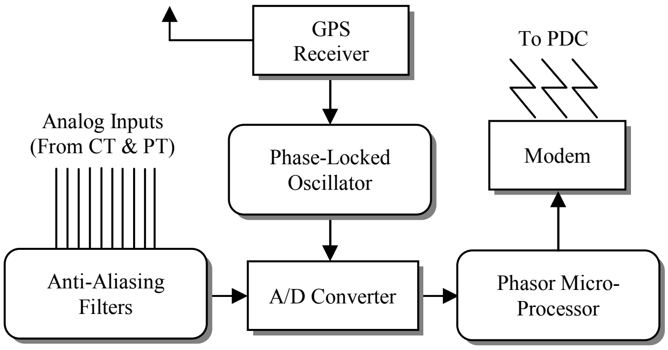

Fig. 1. Basic components of Phasor Measurement Unit

III. PMU PLACEMENT PROBLEM FORMULATION

A PMU placed at a given bus is capable of measuring the voltage phasor of the bus as well as the current phasors for all the lines incident to that bus. The entire system can be made observable by placing PMUs at strategic locations. The objective of a PMU placement problem is to accomplish this task by using a minimum number of PMUs.

The PMU placement problem has been formulated on the following assumptions:

1. PMUs have an infinite channel capacity

2. There is availability of communication at each node

For an n bus system, PMU placement problem can be formulated using Integer Programming method as follows [9]:

$$\min \sum_{i}^{n} w_i . x_i$$
$$\text{subject to } f(\boldsymbol{X}) \geq \boldsymbol{b} \tag{1}$$

where **X** is a binary decision variable vector, whose entries are defined as:

$$x_i = \begin{cases} 1; \text{ if a PMU is installed at bus } i \\ 0; \text{ otherwise} \end{cases} \tag{2}$$

w_i is the cost of PMU installed at bus *i*. If costs of all PMUs are equal, all the entries in ***w*** vector will be 1. In our case, all the entries of w_i are taken as 1. ***b*** is a vector which represents the required redundancy level of measurements for a bus. If any element of matrix is greater than one, it implies that the corresponding bus is observable from more than one direction. Such a system in which each bus is observable more than once is more robust and reliable. Thus, matrix can be used to fulfill the minimum requirements of robustness. In our case, all the entries of ***b*** are taken as 1.

$f(\boldsymbol{X})$ is a vector function, whose entries are non-zero if the corresponding bus voltage is solvable using the given measurement set and zero otherwise. For infinite channel PMUs, $f(\boldsymbol{X})$ can be found as

$$f(\boldsymbol{X}) = \boldsymbol{A} \,.\, \boldsymbol{X} \tag{3}$$

A is the system connectivity matrix. Each row of A corresponds to a bus of the system and each column corresponds to the buses connected to that bus i.e.

$$A_{ij} = \begin{cases} 1; \text{ if } j^{th} \text{ bus is connected to } i^{th} \text{bus OR } i=j \\ 0; \text{ otherwise} \end{cases} \tag{4}$$

Hence, (1) is solved to obtain the solution vector **X** which represents the optimal PMU locations.

A. *Infinite Channel Optimal PMU Placement*

A PMU placed on a bus can measure voltage and voltage angle of the bus as well as the currents and current angles of the branches incident on it. Therefore, by placing a PMU at a bus, voltages of the connected buses can be determined.

A network of 9 Bus Test System as shown in Fig. 2, can be represented by the constraints shown below:

$$f(x) = \begin{cases} x_1+x_2 & \geq 1 \\ x_1+x_2+x_3+x_7 & \geq 1 \\ x_2+x_3+x_4+x_8 & \geq 1 \\ x_3+x_4+x_5+x_9 & \geq 1 \\ x_4+x_5+x_6 & \geq 1 \\ x_5+x_6 & \geq 1 \\ x_2+x_7 & \geq 1 \\ x_3+x_8 & \geq 1 \\ x_4+x_9 & \geq 1 \end{cases} \tag{5}$$

Constraints of (5) shows that for the observability of bus 1, it is necessary to place PMU either at bus 1 or at bus 2 since

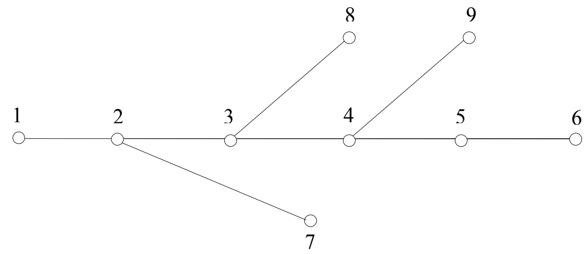

Fig. 2. 9-Bus Test System

bus 2 is connected to bus 1. For observability of bus 2, PMU should be placed at any of the four buses i.e. bus 1, bus 2, bus 3, or bus 7 since buses 1, 3, 7 are connected to bus 2. Similarly for observability of bus 3, PMU should be placed on bus3, bus 4, bus 2 or bus 8 and so on for other buses.

Connectivity matrix ***A*** can be formed as:

$$\mathbf{A} = \begin{array}{c c} & \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \tag{6}$$

The optimal solution of (1) subjected to constraints of (5) is found be bus 2, 3, 4 and 5. Therefore, total of four PMUs are required for full observability of the 9 bus system. In this problem, it is considered that PMU installation at a bus will make all the connected bus observable i.e. there is no channel limit. Hence, this is an infinite channel optimal PMU placement problem.

The algorithm provides optimal solution in reasonable time for systems ranging from few hundred to thousands of buses, mostly transmission networks. For distribution networks, ranging from tens to hundreds of thousand electrical nodes, discussed algorithm takes inappropriately long time due to increase in computational burden. Therefore, it is unsuitable for systems having large number of nodes. Thus, placement problem for large-scale distribution networks demands for low computational requirements to obtain solution in short time.

Hence, it is necessary to develop an algorithm which provides solution by effectively decreasing computational burden and time.

B. *Proposed Algorithm for Optimal PMU Placement*

In practice, distribution systems are mostly radial in nature with primary and secondary feeders (subfeeders & laterals). This concept is presently utilized as a base for development of the new algorithm proposed in this paper. Consider a network of 9 Bus Test System in Fig. 3. The entire network is decomposed into primary and secondary sub-networks. The solution to placement problem is obtained for each sub-network separately. Following are the steps involved:

Step-I

Identification of radial primary network:

For illustrative purpose, the primary network is selected as a path containing maximum number of buses between two visually distant nodes for test systems (although in actual systems, this might differ). For the system under consideration, path containing maximum numbers of buses is from bus 1, 2, 3, 4, 5 to 6, forming a radial network of consecutively connected buses. The selected buses can be represented in vector form as:

$$\mathbf{bus_{prim}} = [1\ 2\ 3\ 4\ 5\ 6]' \tag{7}$$

Step-II

Consideration of branch buses as secondary network:

The remaining buses in connected branches of primary network (in Fig. 3, buses 7, 8 & 9) forms secondary network.

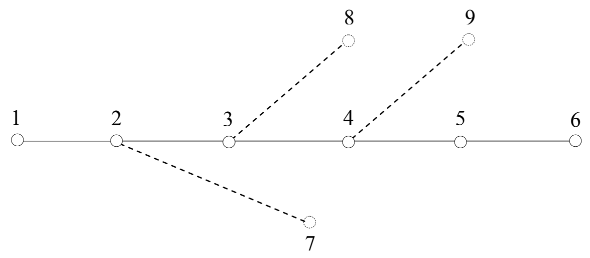

Fig. 3. 9-Bus Test System (Path Containing Maximum Buses)

Step-III

PMU placement in primary and secondary networks:

For proposed algorithm, the solution of placement problem is divided into two stages:

Stage-1

The aim of Stage-1 is to make the primary network completely observable with optimum number of PMUs placed at strategic locations. Stage-1 considers only six buses as a complete network and ignores buses in secondary network. Implementing Binary Integer programming (as discussed in Section III-A), optimal locations are found to be buses 2 and 5.

Identical results can also be obtained by observation without implementing Integer Programming method. This not only saves computational time but also convert optimization problem into a simple mathematical problem, defined by equations below. If $\mathbf{X_1}$ is the solution vector which provides required PMUs for Stage-1, then it is given as:

$$x_{1i} = \begin{cases} 1;\ if\ i = y\,(j) \\ 0;\ otherwise \end{cases} \tag{8}$$

where $\mathbf{Y}$ is a decision vector whose elements represent bus number at which PMUs should be placed. If n_{prim} is total number of primary buses, then length of decision vector, n_Y, is given as:

$$n_Y = (n_{prim}/3)_{\,Quotient} + m \tag{9}$$

where

$$m = \begin{cases} 0;\ if\ n_{prim} is\ completely\ divisible\ by\ 3 \\ 1;\ otherwise \end{cases} \tag{10}$$

In our case, $m = 0$ and $n_Y = 2$. If j=1, 2, 3…, n_Y, then, $\mathbf{Y}$ is given as:

$$y_j = \begin{cases} bus_{prim}(2 + 3(j - 1))\quad ;\ for\ j=1,\ 2,\ ...(n_Y - 1) \\ bus_{prim}(n_{prim} - 1)\ ;\ j=n_Y \end{cases} \tag{11}$$

For discussed bus system:

$$\mathbf{Y} = \begin{bmatrix} 2 \\ 5 \end{bmatrix} \tag{12}$$

and solution vector $\mathbf{X_1}$ corresponding to $\mathbf{Y}$ is found to be:

$$\mathbf{X_1} = [0\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0]' \tag{13}$$

Again, optimal locations are found to be buses 2 and 5.

Hence, it shows that identical results are obtained if we proceed with above discussed method. Since solution vector $\mathbf{X_1}$ can be obtained by solving (8) to (11) only, problem of optimization is converted into a simple mathematical problem. Hence, it reduces computational time.

Stage-2

The aim of Stage-2 is to make the secondary network observable. This stage follows the optimization through integer programming and will proceed as discussed in Section III-A with certain changes and additional constraint. Number of buses to be observed is now reduced which reduces time for obtaining the final solution. It should be noted that PMU placement of Stage-1 makes few secondary buses observable too. Bus 7 is observed by PMU placed at bus 2. If $\boldsymbol{O}$ is observability vector, it can be defined as:

$$\boldsymbol{O} = \boldsymbol{A} * \boldsymbol{X_1} \tag{14}$$

$$o_i = \begin{cases} 1;\ if\ i^{th}\ bus\ is\ observed\ through\ Stage\text{-}1 \\ 0;\ otherwise \end{cases}$$

In our case**:**

$$\boldsymbol{O} = [1\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0]' \tag{15}$$

$\boldsymbol{b}$ is a vector which represents buses that are required to be observed. It is given as:

$$b_i = \begin{cases} 1;\ if\ i^{th}\ bus\ is\ unobserved \\ 0;\ otherwise \end{cases} \tag{16}$$

In our case:

$$\mathbf{b} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1]' \tag{17}$$

To prevent Stage-2 from placing PMUs again at buses of primary network, additional equality constraint is required given as:

$$\boldsymbol{A_{eq}} * \boldsymbol{X_2} = \boldsymbol{b_{eq}} \tag{18}$$

$$s.t. \quad a_{eq}(i) = \begin{cases} 1; \text{ if } i^{th} \text{ bus is primary bus} \\ 0; \text{ otherwise} \end{cases} \tag{19}$$

$$\boldsymbol{b_{eq}} = [0] \tag{20}$$

$\mathbf{X_2}$ is a solution vector for this stage. Solving the binary integer programming problem such that:

$$f(\boldsymbol{X_2}) \geq \boldsymbol{b} \tag{21}$$

$$f(\boldsymbol{X_2}) = \boldsymbol{A} \, . \, \boldsymbol{X_2} \tag{22}$$

with additional equality constraint:

$$\boldsymbol{A_{eq}} * \boldsymbol{X_2} = \boldsymbol{b_{eq}} \tag{23}$$

yields results given as:

$$\mathbf{X_2} = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1]' \tag{24}$$

Optimal locations are found to be buses 8 and 9 for Stage-2.

$\mathbf{X}$ is a final solution vector; a combination of $\mathbf{X_1}$ and $\mathbf{X_2}$ vectors and is given as:

$$\mathbf{X} = [0\ 1\ 0\ 0\ 1\ 0\ 0\ 1\ 1]' \tag{25}$$

Total of four PMUs at buses 2, 5, 8 and 9 are obtained. Same number of PMUs is also obtained by Integer Programming method, though locations are different. The flow chart of the proposed algorithm is given in Fig. 4.

IV. RESULTS AND DISCUSSION

Proposed algorithm is implemented on IEEE-13, 34 and 123 Node Test Feeders. The results obtained are compared with the results of integer programming method in terms of numbers & location of PMUs as well as consumption of time. The code for algorithms has been developed using MATLAB® *R2010a* computational environment and the tests were performed on Intel® Core™ i3 2.4GHz 4GB personal computer.

For the proposed algorithm, entire network is decomposed into primary and secondary sub-networks which reduce the size of PMU placement problem. Placement of Stage-1 reduces the sites for placement of Stage-2 based on assumptions presented in Section-III. Table-I, II and III outlines the locations obtained on implementation of proposed algorithm for each test system and compares it with the optimal locations obtained from

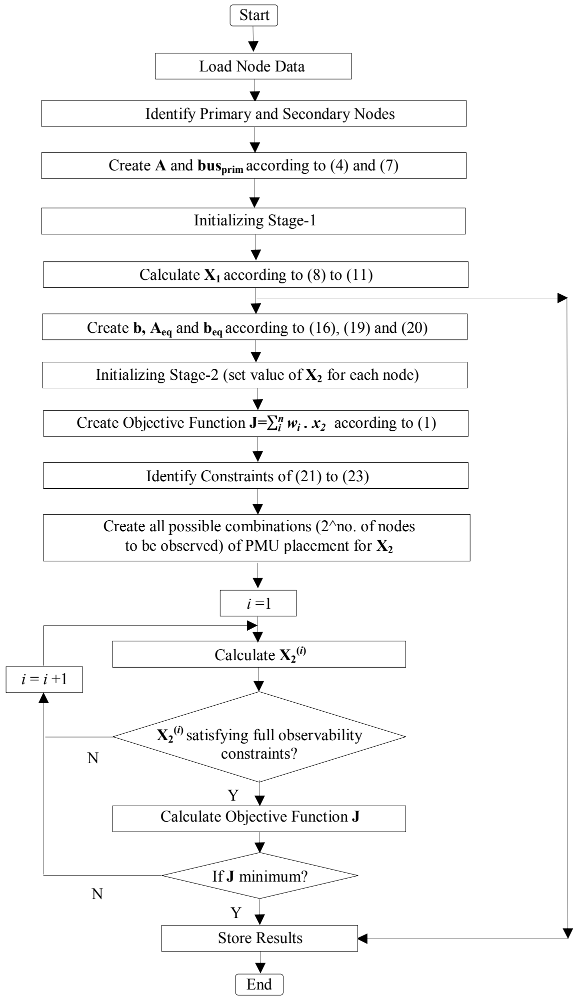

Fig. 4. Flow chart of proposed algorithm

Integer Programming method. Time consumed is also included for both algorithms.

The results show that both algorithms are able to find optimal solution for full observability of a system. Although, the number of locations obtained for both are comparable with each other, the proposed algorithm takes significantly shorter time (approximately half) compared to integer programming method.

It can be inferred that proposed algorithm is more qualified than the Integer Programming method as it may effectively overcome the issue of computational burden and long processing time for practical large-scale distribution systems.

V. CONCLUSION

A simplified and effective approach of PMU placement for large-scale distribution systems is presented in this paper. The proposed algorithm finds an optimal solution to placement problem by decomposition of network into two sub-networks. Since the problem size is reduced due to splitting into two stages, this decomposition reduces the computational time required to obtain optimal locations of PMUs compared to the frequently used Integer Programming method considering the whole network. It can be concluded that the proposed algorithm provides an effective solution to observability of a large-scale distribution system.

PMU placement problem in practical situations involve more than full observability and minimizing cost. Requirements from the specific application should be taken into account, for example influence of PMU malfunction, communication failures, measurement redundancy and uncertainty, number of measurement channels, environmental conditions and existence of other associated devices. These factors may be considered for further investigation and development of placement strategies to generate more practical solutions to placement problem.

TABLE I

OPTIMAL PMU LOCATIONS FOR IEEE-13 NODE FEEDER

Method	Stage	PMU Locations (Node No.)	PMUs	Computa-tional Time(sec)
Proposed Algorithm	1	684, 633	2	0.0285
	2	645, 680, 650, 675	4	
		Total	6	
Integer Programming	-	645, 632, 671 692, 633, 684	6	0.0354

TABLE II

OPTIMAL PMU LOCATIONS FOR IEEE-34 NODE FEEDER

Method	Stage	PMU Locations (Node No.)	PMUs	Computa-tional Time(sec)
Proposed Algorithm	1	836, 858, 854, 824, 814,806, 802	7	0.0413
	2	810, 820, 890, 844, 848, 862	6	
		Total	13	
Integer Programming	-	802, 808, 850, 824, 854, 858, 836, 820, 888, 844, 846, 862	12	0.0847

TABLE III

OPTIMAL PMU LOCATIONS FOR IEEE-123 NODE FEEDER

Method	Stage	PMU Locations (Node No.)	PMUs	Computa-tional Time(sec)
Proposed Algorithm	1	18, 25, 30, 38, 40, 47, 51, 52, 76, 89, 94, 97, 105	13	0.208
	2	1, 4, 6, 12, 14, 15, 20, 22, 24, 32, 33, 37, 43, 46, 55, 56, 58, 60, 64, 66, 69, 70, 74, 78, 82, 84, 88, 92, 96, 100, 103, 107, 110, 113, 135, 149	36	
		Total	49	
Integer Programming	-	1, 4, 6, 12, 14, 15, 19, 22, 23, 29, 31, 33, 37, 39, 41, 43, 46, 47, 51, 54, 55, 56, 59, 60, 64, 66, 69, 71, 74, 76, 78, 82, 85, 88, 90, 91, 96, 97, 100, 103, 105, 107, 110, 114, 135, 152, 250, 150	48	0.523

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